

# A loop quantum multiverse?

Martin Bojowald

*Institute for Gravitation and the Cosmos, The Pennsylvania State University, University Park, PA 16802, USA*

**Abstract.** Inhomogeneous space-times in loop quantum cosmology have come under better control with recent advances in effective methods. Even highly inhomogeneous situations, for which multiverse scenarios provide extreme examples, can now be considered at least qualitatively.

**Keywords:** loop quantum cosmology, effective equations, multiverse, signature change

**PACS:** 98.80.Bp, 98.80.Qc, 04.60.Pp

## INTRODUCTION

The question of whether there is one universe or a collection of different ones in a multiverse is inherently inhomogeneous, and therefore requires any quantum cosmological treatment to go beyond the common minisuperspace constructions. It remains extremely difficult to address in theories such as loop quantum gravity, which do not yet give rise to reliable intuitive and tractable phenomena in anything but the simplest models. Nevertheless, recent progress on effective descriptions of loop quantum gravity has revealed general, perhaps even universal, effects at high curvature, which can be used to test whether the theory makes it likely for the required structures of a multiverse to form. Quite surprisingly, the cosmological scenarios based on these new results are entirely unlike anything that has been imagined in most homogeneous models of cosmology. Some quantum-geometry effects can be so strong at high density that they trigger signature change, an implication which has been overlooked for several years because it can only be seen when inhomogeneity is implemented consistently. Consequences for a possible multiverse are discussed in this article.

## BIG-BANG SINGULARITY

In isotropic loop quantum cosmology [1, 2], the wave function  $\psi_\mu$ , in terms of a geometrical variable  $\mu$  quantizing the spatial volume or the scale factor, can be extended to a universe before the big bang, according to a difference equation of the form

$$C_+(\mu)\psi_{\mu+1} - C_0(\mu)\psi_\mu + C_-(\mu)\psi_{\mu-1} = \hat{H}_{\text{matter}}(\mu)\psi_\mu. \quad (1)$$

Following the recurrence, the wave function is evolved through  $\mu = 0$ , the classical singularity [3, 4]. Departures not only from classical dynamics but also from the continuous Wheeler–DeWitt equation arise because there

are strong “holonomy modifications” at nearly Planckian density, to be discussed in more detail later in this article. These corrections provide the terms by which the difference operator on the left-hand side of (2) differs from a second-order derivative by  $\mu$  as it appears in the Wheeler–DeWitt equation. If finite shifts in the difference operator are Taylor-expanded, a series of higher-order corrections in the momentum of  $\mu$  (a curvature component) is obtained [5]. Holonomy modifications therefore contribute to higher-curvature corrections expected in any quantum theory of gravity, but they lack higher time derivatives and therefore do not provide complete curvature terms.

Motivated by the use of holonomies instead of connection components in the full theory of loop quantum gravity, holonomy modifications replace the Hubble parameter  $H$  by a bounded function  $\sin(\ell H)/\ell$  in a modified Friedmann equation

$$\frac{\sin(\ell H)^2}{\ell^2} = \frac{8\pi G}{3}\rho, \quad (2)$$

with an ambiguity parameter  $\ell$  of the dimension of length (possibly related to the Planck length). Using this modified Friedmann equation, one obtains in simple models an effective picture of singularity resolution given by a bounce [6]: Clearly, the energy density for solutions to (2) must always remain bounded.

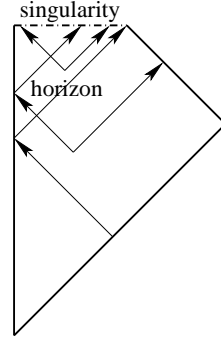
Note that higher-order corrections in an expansion of  $\sin(\ell H)^2/\ell^2 \sim H^2(1 + O(\ell^2 H^2))$  are indeed of the same size as usually expected for higher-curvature corrections, given by the matter density divided by something close to the Planck density. However, (2) ignores higher time derivatives which should be of similar size as they, too, contribute to higher curvature terms. As long as these terms are ignored, one cannot use (2) at high density, near the maximum of the sine function, and it remains unclear whether loop quantum cosmology generically gives rise to bounces as an effective picture of its singularity resolution. In some models with specific matter content (a

free massless scalar or kinetic domination), one can show that higher time derivatives are absent or small [7]. There is therefore a class of models in loop quantum cosmology in which the mechanism of singularity resolution can effectively be described as a bounce, and we will explore this scenario in more detail here. Toward the end of the article, we will comment on the entropy problem [8] which must be addressed in any bounce model.

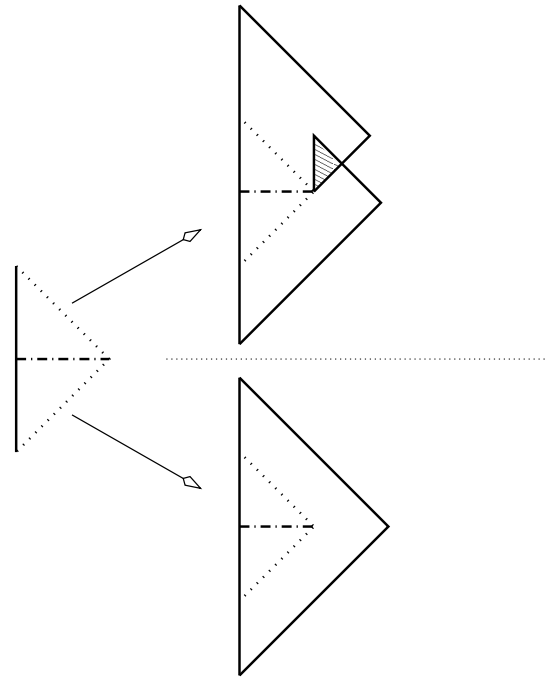
A bounce, in general terms, may give rise to a multiverse picture, using the following line of arguments: Consider a collapsing (part of the) universe. Inhomogeneity builds up as the universe evolves. If space is viewed as a patchwork of nearly homogeneous pieces, their size must be decreased after some time interval to maintain a good approximation, a mathematical process which can be seen as describing the dynamical fractionalization of space. These statements are true in any collapsing universe. If one uses a theory that gives rise to a bounce mechanism, denser patches that reach Planckian density earlier bounce first (assuming that homogeneous models are good for the patch evolution). These bounced patches appear as expanding regions embedded within a still-contracting neighborhood. Given the opposite expansion behaviors, it is difficult to imagine that they can maintain causal contact with their neighborhood. A multiverse picture not unlike that of bubble nucleation in inflation results, except that there is no analog of “bubbles within bubbles” because a patch, once it has bounced, keeps expanding and diluting for a long time and would have to recollapse to trigger new bounces.

Another, independent mechanism that, too, starts with general properties of inhomogeneous collapse uses features of black holes: As inhomogeneity increases, black holes may form and grow. If the singularities they contain classically are resolved in quantum gravity, the question is where this dense space-time region leads to. The interior space-time within the horizon of a Schwarzschild black hole, Fig. 1, can be treated like cosmological models and is resolved just like the big-bang singularity if modifications of loop quantum cosmology are used [9]. Also here, the presence of largely uncomputed higher-curvature corrections means that no effective space-time for a non-singular black hole is known. But one can try to see how a non-singular, possibly bouncing interior could be embedded within an inhomogeneous black-hole space-time. The non-singular interior may be embedded in a spherically symmetric exterior in different ways, shown in Fig. 2. If space-time splits off into a baby universe, a causally disconnected region is obtained, as illustrated in Fig. 3. Multiple such processes provide a multiverse.

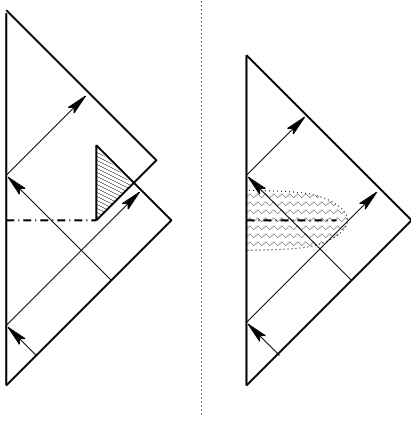
We need good control on inhomogeneity if we want to fill these scenarios with more details. This task is difficult to achieve in non-perturbative quantum gravity, but we can use effective theory to include the key effects in a



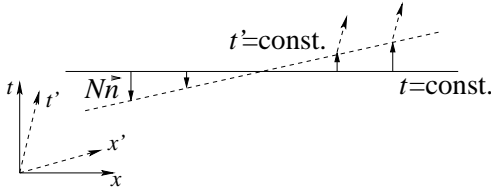
**FIGURE 1.** A classical space-time diagram for black-hole collapse, culminating in a singularity covered by a horizon. If space-time is spherically symmetric, the vacuum part of the interior within the horizon takes on the form of a homogeneous cosmological model. (This figure as well as Figs. 2 and 3 are taken from [10].)



**FIGURE 2.** The vacuum interior region of Fig. 1 can be quantized with methods of loop quantum cosmology, removing the classical singularity. The enlarged, non-singular interior may be embedded in spherically symmetric space-time in two causally different ways. (It is not known at present how precisely to construct such embeddings.) First, the post-singularity interior may open up into a new space-time without causal contact with the original outside region. Secondly, the interior may re-open into the original space-time, spilling out its matter as some kind of black-hole explosion.



**FIGURE 3.** The two alternatives shown in Fig. 2 give rise to different space-time models. A baby universe is obtained from every black hole if the post-singularity interior does not connect causally back to the original space-time. Many such processes would then give rise to a multiverse. If the interior reconnects to the original space-time, black holes are merely compact, extremely dense objects within a single universe.



**FIGURE 4.** Lorentz boosts can be viewed as linear deformations of spatial slices in Minkowski space-time. Here, the standard Minkowski diagram is redrawn with a slightly different viewpoint, focussing on equal-time spatial slices in space-time. Tilted axes show how orthogonality in Minkowski geometry is to be represented after a boost to  $(t', x')$ .

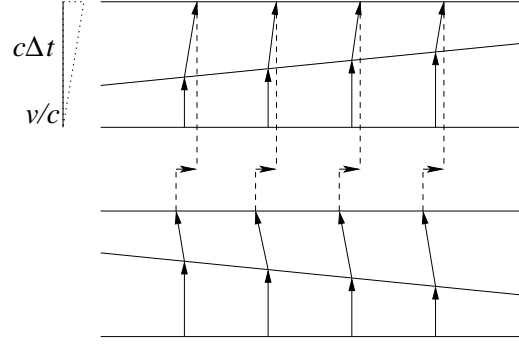
tractable model.

## SPACE-TIME STRUCTURE

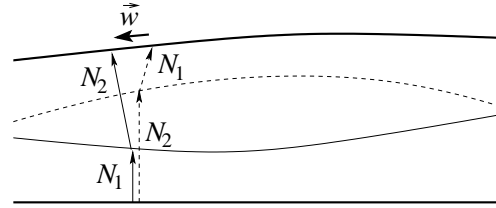
In order to develop ingredients for an effective description of quantum space-time in loop quantum gravity, we begin with a formulation and generalization of the relevant classical structures. Classical space-time has as symmetries Poincaré transformations, which one may view as linear deformations of spatial slices in space-time: we have deformations along the normal  $\vec{n}$  by

$$N(\vec{x}) = c\Delta t + \frac{\vec{v} \cdot \vec{x}}{c}$$

with time translations and boosts, see Fig. 4, or within a slice along  $\vec{w}(\vec{x}) = \Delta\vec{x} + \mathbf{R}\vec{x}$  with spatial translations and rotations.



**FIGURE 5.** The Poincaré algebra follows geometrically from combinations of deformations as in Fig. 4. Shown here is the example of a (boost, time-translation) commutator, with the two orderings shown at the top and bottom. A normal deformation by  $N_1(x) = vx/c$  (Lorentz boost) and one by  $N_2(x) = c\Delta t - vx/c$  (reverse Lorentz boost and waiting  $\Delta t$ ) commute up to a spatial displacement  $w(x) = \Delta x = v\Delta t$ , as computed using the small triangle with angle  $v/c$ .



**FIGURE 6.** The space-time structure of general relativity is obtained by allowing for non-linear deformations of spatial slices. The Poincaré algebra is replaced by the infinite-dimensional hypersurface-deformation algebra. Using classical space-time geometry, one can derive (3)–(5); see e.g. [12].

Algebraic calculations of commutators can be replaced by geometrical pictures, such as the one shown in Fig. 5. In this way, it turns out, one is more open to potential modifications of the algebra due to quantum effects. It is also possible to extend the picture to general relativity without much effort. We simply view local Lorentz transformations or non-linear coordinate changes as non-linear deformations of space, as in Fig. 6. Instead of the well-known commutators of the Poincaré algebra, we obtain the not-so-well-known hypersurface-deformation algebra of infinitely many generators  $(S(\vec{w}(\vec{x})), T(N(\vec{x})))$ , labeled by a vector field  $\vec{w}(\vec{x})$  and a function  $N(\vec{x})$  in space, with [11]

$$[S(\vec{w}_1), S(\vec{w}_2)] = S((\vec{w}_2 \cdot \vec{\nabla})\vec{w}_1 - (\vec{w}_1 \cdot \vec{\nabla})\vec{w}_2) \quad (3)$$

$$[T(N), S(\vec{w})] = T(\vec{w} \cdot \vec{\nabla} N) \quad (4)$$

$$[T(N_1), T(N_2)] = S(N_1 \vec{\nabla} N_2 - N_2 \vec{\nabla} N_1). \quad (5)$$

Hypersurface deformations not only generalize the Poincaré algebra, they also geometrize the dynamics of space-time. As shown by [13, 14], second-order field

equations for metrics, invariant under the hypersurface-deformation algebra, must equal Einstein's. Moreover, invariance under the hypersurface-deformation algebra implies general covariance.

All this is classical physics. The problem of quantum gravity can be approached by asking: How does quantum physics change hypersurface deformations?

## CANONICAL GRAVITY

In order to see how quantum gravity may affect the relations (3)–(5), we must find operators that quantize the classical expressions of  $T(N)$  and  $S(\vec{w})$ . Posing the question in this way suggests that canonical quantum gravity might be closest to answering it, and loop quantum gravity is currently the best-developed canonical approach. In this framework, computing the quantum version especially of (5) in complete detail remains challenging, but a diverse set of methods, including but not restricted to effective techniques, has during the last few years led to mutually consistent and apparently universal results in most of the model systems usually considered in general relativity [15, 16, 17, 18, 19, 20, 21, 22, 23, 24]; see [12, 25] for a general discussion.

Since we will be using methods of loop quantum gravity [26, 27], we describe space-time geometry by a canonical pair of an  $\mathfrak{su}(2)$ -valued “electric field”  $\vec{E}_i$  and a “vector potential”  $A_i$ . (An under-arrow indicates a covariant vector, or a 1-form.) The gravitational electric field is a triad and determines spatial distances and angles by three orthonormal vectors  $\vec{E}_i$ ,  $i = 1, 2, 3$ , at each point in space. The gravitational vector potential  $A_i$  is a combination of different measures of curvature of space: the Ashtekar–Barbero connection.

In quantum field theory, one uses integrated (smeared) fields to construct creation operators by which one can generate all Fock states out of the vacuum. In loop quantum gravity, the geometrical fields offer a natural smearing of  $A_i$  along curves (exponentiated to holonomies) and  $\vec{E}_i$  over surfaces (fluxes). Loop quantum gravity uses holonomies as creation operators to construct a state space. In what follows, we illustrate these objects using a  $U(1)$ -connection  $\vec{A}$  for simplicity. Holonomies are then  $h_e = \exp(i \int_e d\lambda \vec{A} \cdot \vec{t}_e)$  integrated along curves  $e$  in space, with tangent  $\vec{t}_e$ . For every possible  $e$ ,  $\hat{h}_e$  provides excitations of geometry along this curve: As we will see momentarily, surfaces intersected by  $e$  gain area as the excitation level on  $e$  is increased.

To construct the corresponding quantum theory, we start with a basic state  $\psi_0$ ,  $\psi_0(A) = 1$ . Excited states are obtained by acting with holonomies:

$$\psi_{e_1, k_1; \dots; e_i, k_i}(A) = \hat{h}_{e_1}^{k_1} \dots \hat{h}_{e_i}^{k_i} \psi_0(A) \quad (6)$$

$$= \prod_e h_e(A)^{k_e} = \prod_e \exp(ik_e \int_e d\lambda \vec{A} \cdot \vec{t}_e),$$

written in the connection representation. Many excitations along edges are needed to obtain a macroscopic, near-continuum space-time region. Quantum space-time is realized when only a small number of curves  $e$  is geometrically excited, and the action of a single holonomy operator has strong implications on the overall state. Holonomy corrections to the classical equations, as already used in (2), are then significant.

Derivative operators are quantized fluxes  $\int_S d^2 y \vec{n} \cdot \hat{\vec{E}}$  for surfaces  $S$  in space, with co-normal  $\vec{n}$ . They act by

$$\begin{aligned} \int_S d^2 y \vec{n} \cdot \hat{\vec{E}} \psi_{g,k} &= \frac{8\pi G \hbar}{i} \int_S d^2 y \vec{n} \cdot \frac{\delta \psi_{g,k}}{\delta A(y)} \quad (7) \\ &= 8\pi \ell_{\text{Pl}}^2 \sum_{e \in g} k_e \text{Int}(S, e) \psi_{g,k} \end{aligned}$$

with the intersection number  $\text{Int}(S, e)$  and the Planck length  $\ell_{\text{Pl}} = \sqrt{G\hbar}$ . On the right-hand side, we sum only integers, implying a discrete spectrum for fluxes. The same kind of discreteness is realized if we go back to  $SU(2)$ -valued fields, in which case we simply replace derivatives by angular-momentum operators, and integers  $k_e$  by spin quantum numbers [28]. Geometry is discrete: for gravity, fluxes with discrete spectra represent the spatial metric.

## Dynamics

For the discrete dynamics of cosmic expansion, one must quantize the gravitational Hamiltonian (constraint). Since it depends on the connection but only holonomies can be represented as operators in loop quantum gravity, modifications as in (2) are necessary, but now for the full theory [29, 30]. The modified dynamics then takes into account details of how discrete space grows by creating new lattice sites (atoms of space), changing the excitation level of geometry as measured by fluxes.

The classical form of the Hamiltonian is somewhat analogous to that of Yang–Mills theory on Minkowski space-time, where

$$H = \kappa \int d^3 x (|\vec{E}_i|^2 + |\vec{B}_i|^2) \quad (8)$$

for  $\vec{B}_i = \nabla \times A_i + C_{ijk} A_j \times A_k$  (and structure constants  $C_{ijk}$ ). For gravity on any space-time, only showing the crucial terms,

$$H(N) = \frac{1}{16\pi G} \int d^3 x N \frac{\sum_{ijk} \epsilon_{ijk} (\vec{B}_i \times \vec{E}_j) \cdot \vec{E}_k}{\sqrt{\frac{1}{6} |\sum_{ijk} \epsilon_{ijk} (\vec{E}_i \times \vec{E}_j) \cdot \vec{E}_k|}} + \dots \quad (9)$$

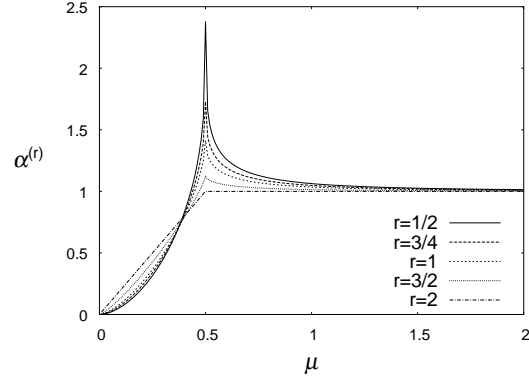
with  $C_{ijk} = \epsilon_{ijk}$ . The presence of a free function  $N$  in (9), as opposed to (8), realizes the freedom of one's choice of time coordinate in generally covariant theories. Indeed,  $H(N)$ , plus matter Hamiltonians, plays the role of the time deformation generator  $T(N)$  introduced earlier. If we can quantize  $H(N)$  and compute commutators of the resulting operators, we can see if and how (5) and the space-time structure it encodes might change. Not surprisingly, the required calculations are rather complicated and remain incomplete, but some results are known.

The form of the Hamiltonian together with properties of the loop representation implies characteristic corrections when quantized. We have already mentioned higher-order corrections resulting from an expansion of holonomies by  $\vec{A}_i$ , used in place of the classical  $\vec{B}_i$  in (9). Holonomy corrections will modify any gravitational Hamiltonian in loop quantum gravity, and therefore indicate that (5) might be quantum corrected. However, holonomy corrections are not the only ones. There is also quantum back-reaction, which is present in any interacting quantum theory and gives rise to higher-time derivatives and curvature corrections. In canonical quantizations, effective techniques provide systematic methods to compute such terms [31, 32, 33]. Note that holonomy corrections and quantum back-reaction (higher-time derivatives) both depend on the curvature. The magnitude of holonomy corrections and quantum back-reaction therefore cannot easily be distinguished, but their algebraic features are sufficiently different from each other to disentangle their implications for quantum space-time, using the substitutes of (5) they imply.

There is a third type of corrections in loop quantum gravity which is easier to handle and which we will discuss first. We obtain inverse-triad corrections from quantizing [34]

$$\left\{ \vec{A}_i, \int \sqrt{|\det E|} d^3x \right\} = 2\pi G \epsilon^{ijk} \frac{\vec{E}_j \times \vec{E}_k}{\sqrt{|\det E|}} \quad (10)$$

whose right-hand side appears in the Hamiltonian (9) but cannot be quantized directly, owing to non-existing inverses of flux operators (7) with discrete spectra containing zero. The left-hand side of (10), on the other hand, can be quantized and is regular, but implies quantum corrections especially for small flux eigenvalues; Fig. 7. These corrections can be computed explicitly in models and provide an automatic cut-off of the  $1/E$ -divergences [35, 36, 37]. They refer to flux eigenvalues in relation to the Planck scale, and are therefore independent of holonomy and higher-curvature corrections which depend on the curvature scale or the energy density. Inverse-triad corrections are therefore more reliable than holonomy corrections, given that higher-curvature terms in loop quantum gravity remain largely unknown.



**FIGURE 7.** Inverse-triad corrections of loop quantum gravity imply a correction function  $\alpha(\mu)$  that depends on ratios  $\mu$  of flux eigenvalues by the Planck area and multiplies any occurrence of inverse triads for instance in Hamiltonians. These functions approach the classical limit  $\alpha = 1$  from above for large fluxes (coarser spatial lattices) and cut off classical divergences by strong quantum corrections at small flux values. The parameter  $r$  describes a quantization ambiguity, which does not affect qualitative features such as  $\alpha$  becoming small near  $\mu = 0$  and  $\alpha$  approaching one from above for  $\mu \rightarrow \infty$ .

## Inverse-triad corrections

For any type of corrections, we can study dynamical implications by inserting their correction functions in the classical Hamiltonian. For inverse-triad corrections, for instance, we have

$$\frac{1}{16\pi G} \int d^3x N \alpha(\vec{E}_l) \frac{\sum_{ijk} \epsilon_{ijk} (\vec{B}_i \times \vec{E}_j) \cdot \vec{E}_k}{\sqrt{\frac{1}{6} |\sum_{ijk} \epsilon_{ijk} (\vec{E}_i \times \vec{E}_j) \cdot \vec{E}_k|}} + \dots \quad (11)$$

with a correction function  $\alpha(\vec{E}_l)$  as in Fig. 7. The Hamiltonian generates time translations as part of the hypersurface-deformation algebra; when the Hamiltonian is modified, the Poisson-bracket algebra is therefore different from the classical one, (3)–(5). By consistency conditions of gravity as a gauge theory, quantum corrections deform but do not violate covariance [15]. We have

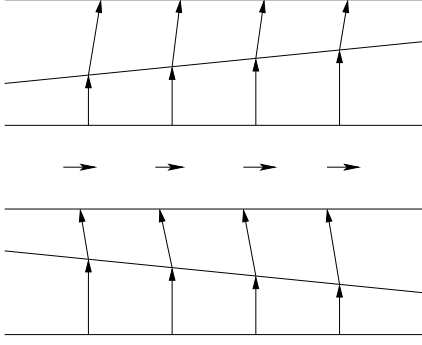
$$[S(\vec{w}_1), S(\vec{w}_2)] = S((\vec{w}_2 \cdot \vec{\nabla}) \vec{w}_1 - (\vec{w}_1 \cdot \vec{\nabla}) \vec{w}_2) \quad (12)$$

$$[T(N), S(\vec{w})] = T(\vec{w} \cdot \vec{\nabla} N) \quad (13)$$

$$[T(N_1), T(N_2)] = S(\alpha^2 (N_1 \vec{\nabla} N_2 - N_2 \vec{\nabla} N_1)). \quad (14)$$

The algebra of hypersurface deformations is deformed, and the laws of motion on quantum space-time change.

Repeating the construction in Fig. 5, but using the deformed algebra as in Fig. 8, the relation of a spatial displacement to the boost velocity is modified:  $\Delta x = \alpha^2 v \Delta t$ . Discrete space speeds up propagation. (According to Fig. 7, we have  $\alpha > 1$  unless we are in strong quantum



**FIGURE 8.** The hypersurface-deformation algebra (14) in the presence of inverse-triad corrections modifies the classical laws of motion. With the constructions in Fig. 5, spatial displacements during a time  $\Delta t$  at velocity  $v$  differ from the classical value:  $\Delta x = \alpha^2 v \Delta t$ . Classical geometry can no longer be used to derive this relation, but it can be computed from the commutator (14) with the functions  $N_1$  and  $N_2$  given in Fig. 5. Since  $\alpha > 1$  in mildly-modified, semiclassical regimes, the displacement is larger than expected classically and velocities seem to increase compared to the expected  $v$ . Even the speed of light is larger than classically, but there is still a consistent causal structure [38] thanks to the closed algebra (14).

regimes of small  $\mu$  in which additional corrections have to be taken into account.)

Modified velocities can also be seen in cosmological perturbation equations, in which the dynamics of density perturbations  $u$  and gravitational waves  $w$  now is [39]

$$-u'' + s(\alpha)^2 \Delta u + (\ddot{z}/\ddot{z})u = 0, \quad (15)$$

$$-w'' + \alpha^2 \Delta w + (\ddot{a}/\ddot{a})w = 0. \quad (16)$$

The function  $\alpha$  that modifies the speed of gravitational waves is shown in Fig. 7, and all other quantum-corrected functions,  $s(\alpha)$ ,  $\ddot{a}$  and  $\ddot{z}$  are known in terms of  $\alpha$  but by rather involved equations. As one general consequence,  $s(\alpha) \neq \alpha$ . We therefore have different speeds for different modes, and obtain corresponding corrections to the tensor-to-scalar ratio as a characteristic signature of deformed space-time. For falsifiability of the theory, it is crucial that  $\alpha - 1$  becomes large for small lattice spacing (flux values), while large lattice spacing implies discretization effects and strong violations of continuum physics. Two-sided bounds on the discreteness scale are obtained, improving the common dimensional expectations (effects of the size of the average density divided by the Planck density) by several orders of magnitude [40, 37].

Before we move on to the other types of quantum corrections, we discuss the conceptual nature of quantum space-time subject to (12)–(14) or related modifications. No effective line element can exist, for the metric and coordinate differentials transform in non-matching ways, the metric — a phase-space function — accord-

ing to modified gauge transformations by (14), but any  $dx$  by standard coordinate changes. One could try to find new differential-geometry structures, such as non-commutative [41, 42] or fractal ones [43], that could provide an invariant line element together with metric coefficients subject to modified gauge transformations. But even without a concrete space-time model, quantum space-time is well-defined because all observables can be computed from (12)–(14) by canonical methods. (See [44] for canonical gravity.) Quantum space-time is also covariant since the full gauge algebra is realized, albeit in a deformed manner [45]. Quantum space-time is just not Riemannian space-time, but this is not to be expected anyway given the presence of discrete structures.

## Quantum back-reaction

Higher-curvature terms in effective quantum gravity [46, 47] modify the classical dynamics, but, unlike quantum-geometry corrections of loop quantum gravity, leave the algebra (5) unchanged [48]: they give theories in which the space-time structure is still classical. They can be derived by standard methods of low-energy effective actions, in which a derivative expansion expresses non-locality by higher time derivatives. Covariance of the theory together with a Poincaré-invariant vacuum state, around which the low-energy effective action expands the quantum dynamics, imply that corrections can only be by space-time scalars, or curvature invariants of increasing order in the derivative expansion.

Loop quantum gravity, as a canonical theory, does not allow easy applications of standard low-energy effective actions which are often based on path integrals. Moreover, it is not clear whether it has a Poincaré-invariant vacuum (or other) state, or whether such a state would be the right base to expand around for, say, quantum cosmological phenomena. It is therefore necessary to use a procedure which is canonical, and at the same time general enough to encompass different quantum states.

Such a procedure [31, 32] can be found by turning Ehrenfest's equations into systematic expansions. Ehrenfest's equations in quantum mechanics express the rates of change of expectation values of basic operators in terms of other, usually more complicated expectation values. For instance, in quantum mechanics of a particle of mass  $m$  in a potential  $V(x)$ , we have

$$\frac{d\langle \hat{x} \rangle}{dt} = \frac{\langle [\hat{x}, \hat{H}] \rangle}{i\hbar} = \frac{\langle \hat{p} \rangle}{m} \quad (17)$$

of the simple classical form. The expectation value of the momentum, however, changes to

$$\frac{d\langle \hat{p} \rangle}{dt} = -\langle V'(\hat{x}) \rangle \quad (18)$$

which for a non-quadratic potential is not a simple expectation value of  $\hat{x}$ . One can formally expand

$$\begin{aligned}\langle V'(\hat{x}) \rangle &= \langle V'(\langle \hat{x} \rangle + (\hat{x} - \langle \hat{x} \rangle)) \rangle \\ &= V'(\langle \hat{x} \rangle) + \frac{1}{2} V'''(\langle \hat{x} \rangle) (\Delta x)^2 + \dots\end{aligned}\quad (19)$$

with the fluctuation  $\Delta x = \sqrt{\langle (\hat{x} - \langle \hat{x} \rangle)^2 \rangle}$  and additional terms that contain higher moments  $\langle (\hat{x} - \langle \hat{x} \rangle)^n \rangle$ .

Since moments of a quantum-mechanical state are degrees of freedom independent of expectation values, the system of equations (17) and (18) is not closed. However, by the same procedure, computing expectation values of commutators as in (17), one can derive new evolution equations for  $\Delta x$  and all other moments. In a semiclassical expansion, only finitely many equations need be taken into account at any fixed order, making the system manageable. Moreover, one can combine this expansion with an adiabatic one in which moments are assumed to vary more slowly than expectation values. With these expansions, as shown in [31], one reproduces the usual low-energy effective action [49] for anharmonic systems. Higher orders in the adiabatic expansion provide higher-derivative terms [33].

The procedure sketched is the right basis to derive effective actions for canonical quantum systems. However, an application to quantum gravity requires additional extensions, most importantly one to include constraints or gauge properties, and to address the problem of time in the absence of an absolute evolution parameter such as the one used in (17). Also this extension is available at the canonical level [50, 51, 52] and allows one to solve the problem of time, at least semiclassically [53, 54, 55]. One could apply these techniques to a quantum version of the algebra (3)–(5), whose symmetry generators in the gauge theory of gravity are constraints. However, a last ingredient required for a successful implementation is still being developed: a generalization of canonical quantum mechanical effective methods to quantum field theories.

At present, it is therefore difficult to include quantum back-reaction in the algebra (5) to see how it could be corrected. One expects higher time derivatives to arise from these corrections, as has been established for quantum-mechanical systems [33], and therefore quantum back-reaction contributes to higher-curvature terms. If the corrections are purely higher curvature, they do not change the hypersurface-deformation algebra. But common low-energy arguments stating that effective quantum back-reaction in gravity is of higher-curvature form assume that there is a Poincaré-invariant vacuum state to be expanded around. In non-perturbative quantum gravity, especially one with a discrete spatial structure such as loop quantum gravity, it is unlikely that there is any exactly Poincaré-invariant state. Standard arguments for

higher-curvature effective actions then break down, and in addition to curvature invariants there may well be other terms that modify the algebra (5), reflecting new space-time structures in the presence of discreteness.

It is possible for quantum back-reaction to modify (5) and compete with the quantum-geometry corrections of loop quantum gravity. Inverse-triad corrections do not refer directly to the density or curvature scale and are therefore safe from competition by quantum back-reaction; they can be discussed separately and bounded observationally. But holonomy corrections always compete with higher-curvature terms.

Any holonomy modification of (5) could, in principle, be undone by modifications due to quantum back-reaction. However, a more detailed look at how quantum back-reaction in constrained systems arises shows that whatever modification may be present, it cannot remove all possible modifications by holonomy corrections. The reason for this is the form of degrees of freedom. Holonomy corrections change even the dependence of Hamiltonian (constraints) on expectation values; they are modifications of the classical dynamics motivated by quantum geometry. Quantum back-reaction gives rise to corrections that depend on moments of a state, as in (19). If one computes Poisson brackets of constraints corrected by quantum back-reaction, all correction terms will still depend on moments after taking the Poisson bracket: Moments are based on polynomial expressions in  $x$  and  $p$ , at least of second degree, and the Poisson bracket of two polynomials of degree at least two is always a polynomial of degree at least two. Quantum corrections of (5) due to quantum back-reaction will therefore contain moments, while those due to holonomy corrections do not.

Quantum back-reaction can cancel holonomy modifications only for special states in which moments are strictly related to expectation values in a specific way. Generically, these corrections, even though their magnitudes are similar, provide different terms that do not cancel each other. Even in the absence of consistent versions of (5) that include quantum back-reaction, it remains meaningful to study holonomy corrections and use their implications for quantum space-time structure, in the same spirit as already discussed for inverse-triad corrections.

## Holonomy corrections and signature change

Holonomy corrections give rise to a difference equation (1) for the wave function, and imply strong modifications at high density. However, quantum back-reaction and higher-curvature corrections are both significant in the same regime, and therefore the high-curvature behavior of loop quantum cosmology remains uncertain.

It is, however, clear that holonomy corrections imply drastic effects on quantum space-time. The hypersurface-deformation algebra with holonomy corrections is not completely known, but all cases that have been computed so far give the same structure [17, 18, 19]:

$$[S(\vec{w}_1), S(\vec{w}_2)] = S((\vec{w}_2 \cdot \vec{\nabla})\vec{w}_1 - (\vec{w}_1 \cdot \vec{\nabla})\vec{w}_2) \quad (20)$$

$$[T(N), S(\vec{w})] = T(\vec{w} \cdot \vec{\nabla} N) \quad (21)$$

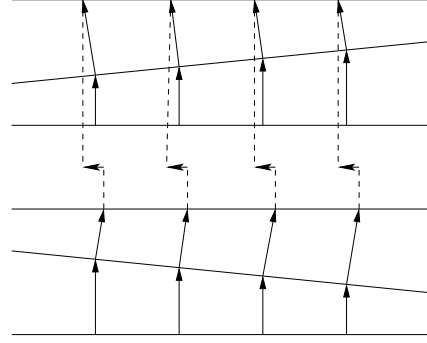
$$[T(N_1), T(N_2)] = S(\beta(N_1 \vec{\nabla} N_2 - N_2 \vec{\nabla} N_1)) \quad (22)$$

with a correction function  $\beta$  that satisfies  $\beta < 0$  at high density (the putative “bounce” of simple models). With modifications as in (2), we have  $\beta = \cos(2\ell H)$  in terms of the Hubble parameter. Notice that inhomogeneities, although they must be present to have a non-trivial derivation of the algebra (22), are not the reason for the modification by  $\beta$ . The reason is holonomy modifications, which already appear for homogeneous background evolution. Inhomogeneity is merely used to show non-trivial space-time effects, as the right-hand side of (20)–(22) vanishes identically for homogeneous  $N$  and  $\vec{w}$ .

A negative  $\beta$ , for instance  $\beta = -1$  at high density, means that constructions such as those in Fig. 5 lead to intransigent motion with  $\Delta x = -v\Delta t$ , a displacement opposite to the velocity. A geometrical interpretation is now more meaningful: A negative  $\beta$  implies that the space-time signature becomes Euclidean [12], as can be seen by comparing Fig. 5 with the Euclidean version Fig. 9. Indeed, if one computes cosmological perturbation equations analogous to (15) and (16), as done in [19, 56], the positive  $\alpha^2$  is replaced by  $\beta$ , giving rise to an elliptic differential equation when  $\beta < 0$ . (Holonomy corrections, so far, do not lead to different parameters  $\beta$  and  $s(\beta)$  for the independent modes of cosmological perturbation equations.) The same effect happens when inhomogeneity is treated non-perturbatively in spherically symmetric models [17], where it can be shown to be largely insensitive to quantization ambiguities [12]. With signature change at high density, the cosmological scenario of loop quantum cosmology is not a bounce, but is reminiscent of the Hartle–Hawking picture, now derived as a consequence of quantum space-time structure in the presence of holonomy corrections.

## MULTIVERSE?

In view of these new results, we must revise our multiverse scenario sketched in the beginning of this article, based on cosmological bounces. (See also [57].) We observed that inhomogeneous collapse combined with a transition to expansion (a “bounce”) may lead to causally disconnected regions, expanding within a larger multiverse. Inhomogeneity of this type is extremely hard to



**FIGURE 9.** Holonomy corrections at high density imply a sign reversal in the correction function of (22). The constructions of Fig. 5 would indicate  $\Delta x = -v\Delta t$ , with reversed displacement arrows. A better geometrical explanation of the reversed sign and arrows is signature change: Redrawing Fig. 5 with right angles according to Euclidean geometry, as shown here, implies the same reversal of spatial-displacement arrows as follows from a reversed sign in (22).

control with present-day non-perturbative quantum gravity, but good effective methods are now available to help us understand the relevant space-time structure. Loop quantum gravity, it turns out, implies radical modifications at Planckian densities, with a quantum version of 4-dimensional Euclidean space instead of space-time.

In Euclidean space, initial-value problems are ill-posed and there is no propagation of structure from collapse to expansion, as assumed in bounce models. Expanding patches that may result are causally disconnected not just from their surrounding space-time, but also from their predecessor which was collapsing. Instead of a bounce, loop quantum cosmology, once inhomogeneity is taken into account consistently, gives rise to a non-singular beginning of the expanding Lorentzian phase we can observe. The transition from Euclidean to Lorentzian signature, when  $\beta = 0$ , is a natural place to pose initial conditions, for instance for an inflaton state. These initial values are unaware of what happened in the collapse phase, so that the picture of dense collapsing patches bouncing first is not realized.

The new signature-change of loop quantum cosmology shares with bounces the combination of collapse with expansion, but the collapse phase does not deterministically affect the expansion phase. As a consequence, there is no entropy problem because no complete information is transmitted through high densities. And yet, the model is non-singular [58].

One may still view the possible collection of expanding universes within one space(-time), combining Euclidean and Lorentzian pieces, as a multiverse. However, any causal contact realized is even weaker than what is usually possible in multiverses, and it may seem more appropriate to talk of separate universes instead of one



however connected larger structure. Each of these expanding patches has its own beginning when space-time emerges by signature change from 4-dimensional space, giving it a clear status as a universe of its own.

## ACKNOWLEDGEMENTS

I am grateful to Mariusz Dabrowski and the organizers of the conference Multicosmofun '12 for their invitation to give a talk on which this article is based. This work was supported in part by NSF grant PHY0748336.

## REFERENCES

1. M. Bojowald, *Living Rev. Relativity* **11**, 4 (2008), <http://www.livingreviews.org/lrr-2008-4>, gr-qc/0601085.
2. M. Bojowald, *Quantum Cosmology: A Fundamental Theory of the Universe*, Springer, New York, 2011.
3. M. Bojowald, *Phys. Rev. Lett.* **86**, 5227–5230 (2001), gr-qc/0102069.
4. M. Bojowald, *Class. Quantum Grav.* **19**, 2717–2741 (2002), gr-qc/0202077.
5. M. Bojowald, *Class. Quantum Grav.* **18**, L109–L116 (2001), gr-qc/0105113.
6. A. Ashtekar, T. Pawłowski, and P. Singh, *Phys. Rev. D* **73**, 124038 (2006), gr-qc/0604013.
7. M. Bojowald, *Phys. Rev. D* **75**, 081301(R) (2007), gr-qc/0608100.
8. R. C. Tolman, *Relativity, Thermodynamics and Cosmology*, Clarendon Press, Oxford, 1934.
9. A. Ashtekar, and M. Bojowald, *Class. Quantum Grav.* **23**, 391–411 (2006), gr-qc/0509075.
10. M. Bojowald, *Once Before Time: A Whole Story of the Universe*, Knopf, New York, 2011.
11. P. A. M. Dirac, *Proc. Roy. Soc. A* **246**, 333–343 (1958).
12. M. Bojowald, and G. M. Paily, *Phys. Rev. D* **86**, 104018 (2012), arXiv:1112.1899.
13. S. A. Hojman, K. Kuchař, and C. Teitelboim, *Ann. Phys. (New York)* **96**, 88–135 (1976).
14. K. V. Kuchař, *J. Math. Phys.* **15**, 708–715 (1974).
15. M. Bojowald, G. Hossain, M. Kagan, and S. Shankaranarayanan, *Phys. Rev. D* **78**, 063547 (2008), arXiv:0806.3929.
16. M. Bojowald, J. D. Reyes, and R. Tibrewala, *Phys. Rev. D* **80**, 084002 (2009), arXiv:0906.4767.
17. J. D. Reyes, *Spherically Symmetric Loop Quantum Gravity: Connections to 2-Dimensional Models and Applications to Gravitational Collapse*, Ph.D. thesis, The Pennsylvania State University (2009).
18. A. Perez, and D. Pranzetti, *Class. Quantum Grav.* **27**, 145009 (2010), arXiv:1001.3292.
19. T. Cailleteau, J. Mielczarek, A. Barrau, and J. Grain, *Class. Quant. Grav.* **29**, 095010 (2012), arXiv:1111.3535.
20. A. Kreienbuehl, V. Husain, and S. S. Seahra, *Class. Quantum Grav.* **29**, 095008 (2012), arXiv:1011.2381.
21. A. Henderson, A. Laddha, and C. Tomlin (2012), arXiv:1204.0211.
22. A. Henderson, A. Laddha, and C. Tomlin (2012), arXiv:1210.3960.
23. C. Tomlin, and M. Varadarajan (2012), arXiv:1210.6869.
24. M. Varadarajan (2012), arXiv:1210.6877.
25. M. Bojowald, *Class. Quantum Grav.* **29**, 213001 (2012), arXiv:1209.3403.
26. C. Rovelli, *Quantum Gravity*, Cambridge University Press, Cambridge, UK, 2004.
27. T. Thiemann, *Introduction to Modern Canonical Quantum General Relativity*, Cambridge University Press, Cambridge, UK, 2007, gr-qc/0110034.
28. C. Rovelli, and L. Smolin, *Phys. Rev. D* **52**, 5743–5759 (1995).
29. C. Rovelli, and L. Smolin, *Phys. Rev. Lett.* **72**, 446–449 (1994), gr-qc/9308002.
30. T. Thiemann, *Class. Quantum Grav.* **15**, 839–873 (1998), gr-qc/9606089.
31. M. Bojowald, and A. Skirzewski, *Rev. Math. Phys.* **18**, 713–745 (2006), math-ph/0511043.
32. M. Bojowald, and A. Skirzewski, *Int. J. Geom. Meth. Mod. Phys.* **4**, 25–52 (2007), proceedings of “Current Mathematical Topics in Gravitation and Cosmology” (42nd Karpacz Winter School of Theoretical Physics), Ed. Borowiec, A. and Francaviglia, M., hep-th/0606232.
33. M. Bojowald, S. Brahma, and E. Nelson, *Phys. Rev. D* **86**, 105004 (2012), arXiv:1208.1242.
34. T. Thiemann, *Class. Quantum Grav.* **15**, 1281–1314 (1998), gr-qc/9705019.
35. M. Bojowald, *Phys. Rev. D* **64**, 084018 (2001), gr-qc/0105067.
36. M. Bojowald, H. Hernández, M. Kagan, and A. Skirzewski, *Phys. Rev. D* **75**, 064022 (2007), gr-qc/0611112.
37. M. Bojowald, G. Calcagni, and S. Tsujikawa, *JCAP* **11**, 046 (2001), arXiv:1107.1540.
38. M. Bojowald, and G. Hossain, *Phys. Rev. D* **77**, 023508 (2008), arXiv:0709.2365.
39. M. Bojowald, and G. Calcagni, *JCAP* **1103**, 032 (2011), arXiv:1011.2779.
40. M. Bojowald, G. Calcagni, and S. Tsujikawa, *Phys. Rev. Lett.* **107**, 211302 (2011), arXiv:1101.5391.
41. A. Connes, *C.R. Acad. Sci. Paris* **323**, 1231–1235 (1996).
42. S. Doplicher, K. Fredenhagen, and J. E. Roberts, *Commun. Math. Phys.* **172**, 187–220 (1995), hep-th/0303037.
43. G. Calcagni, *Phys. Rev. Lett.* **104**, 251301 (2010), arXiv:0912.3142.
44. M. Bojowald, *Canonical Gravity and Applications: Cosmology, Black Holes, and Quantum Gravity*, Cambridge University Press, Cambridge, 2010.
45. M. Bojowald, and G. M. Paily (2012), arXiv:1212.4773.
46. C. P. Burgess, *Living Rev. Relativity* **7** (2004), <http://www.livingreviews.org/lrr-2004-5>, gr-qc/0311082.
47. J. F. Donoghue, *Phys. Rev. D* **50**, 3874–3888 (1994), gr-qc/9405057.
48. N. Deruelle, M. Sasaki, Y. Sendouda, and D. Yamauchi, *Prog. Theor. Phys.* **123**, 169–185 (2009), arXiv:0908.0679.
49. F. Cametti, G. Jona-Lasinio, C. Presilla, and F. Toninelli,

“Comparison between quantum and classical dynamics in the effective action formalism,” in *Proceedings of the International School of Physics “Enrico Fermi”, Course CXLIII*, IOS Press, Amsterdam, 2000, pp. 431–448, [quant-ph/9910065](#).

50. M. Bojowald, B. Sandhöfer, A. Skrzewski, and A. Tsobanjan, *Rev. Math. Phys.* **21**, 111–154 (2009), [arXiv:0804.3365](#).
51. M. Bojowald, and A. Tsobanjan, *Phys. Rev. D* **80**, 125008 (2009), [arXiv:0906.1772](#).
52. M. Bojowald, and A. Tsobanjan, *Class. Quantum Grav.* **27**, 145004 (2010), [arXiv:0911.4950](#).
53. M. Bojowald, P. A. Höhn, and A. Tsobanjan, *Class. Quantum Grav.* **28**, 035006 (2011), [arXiv:1009.5953](#).
54. M. Bojowald, P. A. Höhn, and A. Tsobanjan, *Phys. Rev. D* **83**, 125023 (2011), [arXiv:1011.3040](#).
55. P. A. Höhn, E. Kubalova, and A. Tsobanjan, *Phys. Rev. D* **86**, 065014 (2012), [arXiv:1111.5193](#).
56. T. Caillateau, A. Barrau, J. Grain, and F. Vidotto, *Phys. Rev. D* **86**, 087301 (2012), [arXiv:1206.6736](#).
57. J. Mielczarek (2012), [arXiv:1212.3527](#).
58. M. Bojowald, and G. M. Paily, *Class. Quantum Grav.* **29**, 242002 (2012), [arXiv:1206.5765](#).